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Final Report

Name of the researcher	Sara Moradi
Selection Year	2013
Host institution	ULB
Supervisor	Dr. Bernard Knaepen
Period covered by this report	from 01/03/2016 to 31/12/2016
Title of the project	Multi-scale turbulent transport in fusion plasmas, theoretical developments and data analysis.

1. Objectives of the proposal (1 page)

The international effort to develop fusion energy is on the threshold of a new era. ITER (http://www.iter.org) aims to demonstrate the scientific and technological feasibility of magnetic fusion power. This is a dramatic step in establishing the potential of fusion energy.

In this context, turbulence is one of the most fundamental questions in physics and fusion plasmas provide one of the most challenging environments for its study.

This research project aims to study the turbulent transport phenomena in magnetically confined fusion plasmas and to develop analytical models and numerical tools in order to explain the experimental observations, improve predictive capabilities of the long-time behavior of fusion plasmas, and in particular, self-consistent co-evolution of turbulence and macroscopic profiles (i.e. radial profiles of temperature, density, rotation etc.).

2. Methodology in a nutshell (1 page)

In my work, I studied the turbulent driven transport in fluids and magnetized plasmas using various methodologies from simple Monte-Carlo particle simulations to high-power computing with Gyro-Kinetic code GYRO https://fusion.gat.com/theory/Gyrodoc.

In the following I will present the results of the performed projects (prepared/published papers).

3. Results (8-10 pages)

a) Charge particle motion in the presence of non-Gaussian, Levy, electrostatic fluctuations:

There is a considerable amount of experimental evidence [1-7] and numerical gyrokinetic [8,9] and fluid [10] simulations that indicate that plasma turbulent transport in tokamaks is, under some conditions, non-diffusive. There are several reasons for the possible breakdown of the standard diffusion paradigm which is based on restrictive assumptions including locality, Gaussianity, lack of long range correlations, and linearity. Different physical mechanisms can generate situations where e.g., locality and Gaussianity may be incorrect assumptions for understanding transport. For example, interactions with external fluctuations may introduce long-range correlations and/or anomalously large particle displacements. The source of the external fluctuations could be that not all relevant physics is taken into account such as coherent modes or other non-linear mechanisms. The emergence of such strange kinetics has been studied previously [11-18], e.g., using different modelling strategies where it may be generated by accelerated or sticky motions along the trajectory of the random walk.

In addition, turbulence intermittency is characterized by patchy spatial structures that are bursty in time and coupling to these modes introduces long range correlations and/or Lévy distributed noise characteristics. The probability density functions (PDF) of intermittent events often show unimodal structure with "elevated" tails that deviate from Gaussian predictions [19-21]. Experimental evidence of Lévy statistics in the electrostatic fluctuation at the plasma edge was presented in Ref. [21], with a Lévy index in the range $\alpha = 1.1 - 1.3$ at short times and in the range $\alpha = 1.8-2$ at long times. Further- more, in Ref. [22] it was observed that moving from the inner to the outer region of edge plasma, the Lévy index decreases, suggesting that the PDFs of the turbulence near the boundary region of Heliotron J are nearly Gaussian, whereas at the outer regions of plasma they become strongly non-Gaussian. The statistics of the measured fluctuations at the edge of Stellarators such as Uragan 3M and HELIOTRON J have been observed to change from Lévy to Gaussian at the L to H-mode transition [23-25]. These types of observations are not limited to fusion plasmas, Lévy-type turbulent random processes and related anomalous diffusion phenomena have been observed in a wide variety of complex systems such as semiconductors, glassy materials, nano-pores, biological cells, and epidemic spreading [26] . The kinetic descriptions which arise as a consequence of averaging over the well-known Gaussian statistics seem to fall short in describing the apparent randomness of these dynamical chaotic systems. Thus, the problem of finding a proper kinetic description for such complex systems is a challenge.

Lévy statistics [27] describing fractal processes (Lévy index α where $0 < \alpha < 2$) lie at the heart of complex processes such as anomalous diffusion. Lévy statistics can be generated by random processes that are scale-invariant with anomalous scaling exponents. This means that a trajectory lacks a unique characteristic scale that dominates the process. Geometrically this implies the fractal property that a trajectory, viewed at different resolutions, will exhibit self-similar properties. Indeed, self- similar analysis of fluctuation measurements by Langmuir probes in different fusion devices such as spherical tokamak, reversed field pinch, stellarator, and several tokamaks, have provided evidence to support the idea that density and potential fluctuations are distributed according to Lévy statistics. Furthermore, the experimental evidence of the wave-number spectrum characterised by power laws over a wide range of wave-numbers can be directly linked to the values of Lévy index α of the PDFs of the underlying turbulent processes.

In a previous study [18] the aim was to shed light on the non-extensive properties of the velocity space statistics and characterization of the fractal processes limited to the Fractional Fokker-Planck Equation in terms of Tsallis statistics. The goal of this paper is to study the statistics of charged particle motion in the presence of α- stable Lévy fluctuations in an external magnetic field and linear friction using Monte Carlo numerical simulations. The Lévy noise is introduced to model the effect of non- Gaussian, intermittent electrostatic fluctuations. The statistical properties of the velocity moments and energy for various values of the Lévy index α are investigated as well as the role of Lévy fluctuations on the statistics of the particles' Larmor radii in order to examine potential limitations of gyro-averaging. Fractional kinetics of charged particle transport in a constant parallel magnetic field and a random electric field was studied in Ref. [15]. Going beyond this work, we perform 3-dimensional simulations in a helical magnetic field and study the statistics of the spatial displacements and Larmor radius which were not discussed in Ref. [15] whose numerical results were limited to 2-dimensions using a different type of isotropic Lévy processes. However, memory effects are neglected since the Lévy noise is taken as white or delta correlated in time.

We consider the motion of charged particles in a 3- dimensional magnetic field in a cylindrical domain in the presence of linear friction modelling collisional Coulomb drag and a stochastic electric field according to the Langevin equations

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \quad (1)$$

$$\frac{d\mathbf{v}}{dt} = \frac{q_s}{m_s}\mathbf{v} \times \mathbf{B} - \nu\mathbf{v} + \frac{q_s}{m_s}\mathcal{E}, \quad (2)$$

where q_S and m_S are the charge and mass of the particle species s, ν is the friction parameter and E is a 3-dimensional, homogeneous, isotropic turbulent electric field modelled as an stationary, uncorrelated stochastic process without memory following an α –stable distribution, f(α,β,σ,η), with characteristic expo- √ nent 0<α≤2, skewness β=0, variance σ=1/ 2, and mean η = 0. Here, we use the definition of f(α,β,σ,η) as described in Refs. [28-30].

A periodic straight cylindrical domain with period $L = 2\pi R_0$ is considered, with R_0 being the major radius, and we use cylindrical coordinates (r, θ, z). magnetic field is a helical field of the form,

$$\mathbf{B}(r) = B_\theta(r)\,\hat{\mathbf{e}}_\theta + B_z\hat{\mathbf{e}}_z. \quad (3)$$

A constant magnetic field in z-direction, $B_Z = B_0$, is assumed. The shear of the helical magnetic field, i.e. the dependence of the azimuthal rotation of the field as function of the radius, is determined by the q-profile, $q(r) = rB_Z/(R_0B_\theta)$, where

$$B_\theta(r) = \frac{B(r/\lambda)}{1+(r/\lambda)^2}, \quad (4)$$

for which the q profile is

$$q(r) = q_0\left(1+\frac{r^2}{\lambda^2}\right). \quad (5)$$

In terms of the flux variable,

$$\psi = \frac{r^2}{2{R_0}^2}, \quad (6)$$

q is a linear function of ψ. The numerical integration of Eqs. (1) and (2) is per-

formed using a Runge-Kutta 4th order scheme (RK4) over the interval [0,T]. The time step for the RK4 integration is defined by partitioning the interval [0,T] into N subintervals of width δ = T/N > 0,

$$0 = \tau_0 < \tau_1 < \cdots < \tau_i < \tau_N = T, \quad (7)$$

with the initial conditions r_0, and v_0. We compute r_i and v_i for the subintervals with the time step of dt = δ/n, and at every δ, we include the cumulative integral of the stochastic process using

$$d\mathbf{r}_i = \mathbf{v}_i dt \quad (8)$$

$$d\mathbf{v}_i = \left[\frac{q_s}{m_s}\mathbf{v}_i \times \mathbf{B} - \nu\mathbf{v}_i\right] dt + \mathbf{W} \quad (9)$$

where

$$\mathbf{W} = \frac{q_s}{m_s}\chi \sum_{\delta}(dt)^{(1/\alpha)}\mathcal{E}. \qquad (10)$$

Here, using spherical coordinates, random samples in the E_ρ radial direction are generated with the α-stable random generator developed in Ref. [28-30], and two uniformly distributed angles θ and φ between [0, 2π] are used. In Cartesian coordinates the components of the electric field are $E_X = E_\rho \sin\theta\cos\varphi$, $E_Y = E_\rho \sin\theta\sin\varphi$, and $E_Z = E_\rho \cos\theta$. $N_p = 10^4$ particles are considered, and the simulation time is $T = 500/\tau_C$ where $\tau_C = 2\pi/\Omega_C$ and $\Omega_C = |q_S|B_0/m_S$ is the gyration frequency. We explore the dependence of the particle motion on the index α of the Lévy fluctuations and the parameter ε = χ/ν where χ is the amplitude of the fluctuations and ν is the damping coefficient. The convergence in probability of Lévy driven stochastic differential equations 1 and 2 have been discussed in Ref. [31] where a criteria is established.

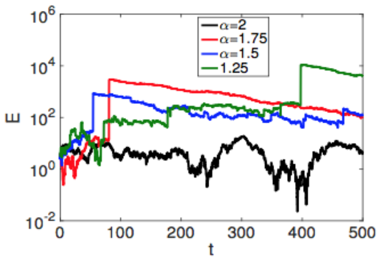

FIG. 1. Samples of normalised particle energy, $E = \frac{1}{2}(v_x^2 + v_y^2 + v_z^2)/v^2(0)$, vs time for different values of $\alpha = 2$ (black), 1.75 (red), 1.5 (blue), and 1.25 (green). Here, $\epsilon = 100$.

Figure 1 shows samples of the particles' energy as function of time for several values of α. It is observed that as α decreases, the random walk in energy is strongly influenced by outlier events which result in intermittent behaviour with appearance of Lévy flights between periods of small perturbations. The rate and the amplitude of the intermittent jumps in energy increase significantly as α is decreased. This behaviour is clearly observed in Figs. 2(a) and (b) where the PDF of Log_{10} of the particle energy, E, and the q = 1/2-moment of the energy as functions of time are shown. As seen in Fig. 2(a) the decay of the PDFs changes from exponential in the case of a Gaussian process to power law in the case of a Lévy process. The power law exponent decreases as α is decreased indicating the increase in the probability of the occurrence of Lévy flights. A breakup in the symmetry of the PDFs is also observed with a shift towards higher values of the energy, as the Lévy index α is reduced. Note that the numerical results indicate that the PDFs relax towards stationary states. The q = 1/2-moments of the energy converge in the considered simulation time span, and there exist about two orders of magnitude increase in the converged values as α varies from a Gaussian pro- cess (α = 2) towards a strongly Lévy distributed process (α = 1.25), as can be seen in Fig. 2(b).

We have performed Monte Carlo numerical simulations of charged particle motion in the presence of a fluctuating electric field obeying non-Gaussian Lévy statistics in a constant magnetic field and linear friction modelling the effect of collisional Coulomb drag. The Lévy noise was introduced in order to model the effect of non-local transport due to fractional diffusion in velocity space resulting from intermittent electrostatic turbulence. The statistical properties of the velocity moments and energy for various values of the Lévy index α were investigated, and the role of Lévy fluctuations on the particles Larmor radii, and the statistical moments of displacements were explored. We observed that as α is decreased, the random walk in energy is strongly influenced by outlier events which result in intermittent behaviour with appearance of Lévy flights in between periods of small perturbations. The rate and the amplitude of the intermittent jumps in energy increases significantly as α is decreased. The PDFs of the particles' Larmor radii change from an exponential decay to a power law decay when the stochastic electrostatic process is changed from a Gaussian to a Lévy process. The power law decay de- creases with decreasing α. This corroborates the findings in Ref. 18 that the q-moment is an appropriate metric characterizing Lévy distributed processes. Our findings suggest that when turbulent electrostatic fluctuations exhibit non-Gaussian Lévy statistics, gyro-averaging and guiding centre approximations may not be fully justified and full particle orbit effects should be taken into ac- count. The results presented here point out potential limitations of gyro-averaging. Turbulent plasmas exhibit a very large range of spatio-temporal scales. To over- come the computational challenge that this implies, it is

FIG. 5. The linear fits of the power law decay (a) for energy, μ_E, and (b) for the Larmor radius, μ_ρ, as functions of the Lévy index α, for different values of ε = 100 (black line with circle symbols), ε = 10 (red line with square symbols).

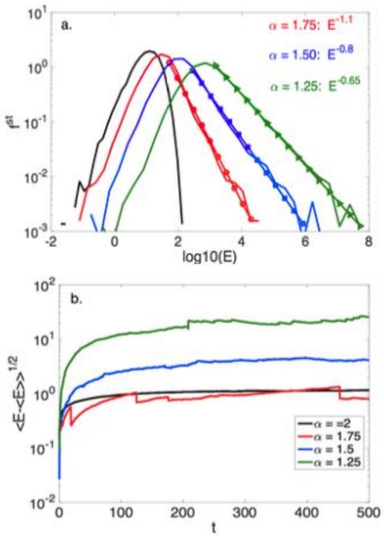

FIG. 2. (a) The steady state PDFs (f^{st}) of $Log_{10}(E)$, and linear fits for the energy decay are shown by lines with symbols, (b) the q-moment are shown with $q = 1/2$ for different values of $\alpha = 2$ (black), 1.75 (red), 1.5 (blue), and 1.25 (green). Here, $\epsilon = 100$.

customary to use reduced descriptions based on spatial and/or temporal averaging of degrees of freedom that evolve on small spatial scales and/or fast time scales compared to the macroscopic scales of interest. For ex-ample, the extensively used gyro-kinetic models assume $\rho_L/L \ll 1$ where ρ_L is the Larmor radius and L is the tokamak minor radius or a characteristic density gradient scale. However, it is important to keep in mind that in a turbulent plasma the Larmor radius is a statistical quantity, $\langle\rho_L\rangle$, (where $\langle\cdot\rangle$ denotes ensemble average) and not an absolute number. For plasmas in Maxwellian equilibrium this issue might not be critical since the probability density function (PDF) of Larmor radii is sharply peaked around the thermal Larmor radius. However, when the PDF exhibits slowly decaying tails due to a significant number of outliers (i.e., particles with anomalously large Larmor radii) the situation is much less trivial. In particular, in the case of algebraic decaying PDFs, statistical moments might not exist and as a result in might not be possible to associate a characteristic scale to the process. The study of scale free stochastic processes has been a topic of significant interest in basic and applied sciences in general and in plasma physics in particular, see for example Refs. [23,33] and references therein. Our numerical results indicate that when the electrostatic fluctuations follow Lévy statistics with index α, the PDFs of Larmor radii exhibit algebraic decay and this might compromise the meaning of $\langle\rho_L\rangle$. Formally, if the PDF of $x \in (0,\infty)$ decays as $f \sim x^{-\mu}$, then the n-th moment, i.e. $\langle x^n\rangle = \int^{\infty} x^n f\, dx$, will diverge, and thus will not be 0 well-defined, for $\mu < n + 1$. Based on this, for $\alpha < 1.75$, $\langle\rho_L\rangle$ is strictly speaking not well-defined. In practice, the mean values might not diverge because the numerically computed PDFs have a cut-off due to limited statistical sampling. However, as the case $\alpha = 1.25$ the fact that $\mu < 2$ implies that the convergence of $\langle\rho_L\rangle$ might be questionable. Also, we would like to note that, in this work as a first step we have limited attention to the study of electrostatic turbulent fluctuations driven by uncorrelated stochastic processes in the absence of memory. However, memory and correlations might play an important role. For example, in Ref. [10] it was shown that non-Markovian effects are present in fluid models of plasma turbulent transport and as a consequence, in this case, effective models of particle transport should include both spatial jumps driven by Lévy processes and memory effects driven by non-Markovian waiting times. On the other hand, the work in Ref. [9] showed that correlations play a role on gyro-kinetic turbulent transport in the presence of shear flows and thus, in this case, the proper treatment requires the use of correlated non- Gaussian random processes. The work presented here could be extended to include memory effects by incorporating non-Markovian statistics in the Monte-Carlo simulation, and also by including correlations using fractional Levy motion models. Note that doing this would naturally introduce a characteristic time scale into the turbulence fluctuation model, e.g. the correlation time or the memory time scale. A problem of interest would then be to study the dependence of the results on these fluctuation time scales and the gyro-period of the orbits. These are interesting problems that we plan to address in the future.

[1]N. Lopez Cardozo, Plasma Phys. Controlled Fusion 37, 799 (1995).

[2]K. W. Gentle, R. V. Bravenec, G. Cima, H. Gasquet, G. A. Hallock, P. E. Phillips, D. W. Ross, W. L. Rowan, and A. J. Wootton, Phys. Plasmas 2, 2292 (1995).

[3]J. D. Callen and M. W. Kissick, Plasma Phys. Controlled Fusion 39, B173 (1997).

[4]P. Mantica, P. Galli, G. Gorini, G. M. D. Hogeweij, J. de Kloe, and N. J. Lopes Cardozo, Phys. Rev. Lett. 82, 5048 (1999).

[5]B. Ph. van-Milligen, E. de la Luna, F. L. Tabars, E. Ascasbar, T. Estrada, F. Castejn, J. Castellano, I. Garcia-Cortes, J. Herranz, C. Hidalgo, J. A. Jimenez, F. Medina, M. Ochando, I. Pastor, M. A. Pedrosa, D. Tafalla, L. Garca, R. Sanchez, A. Petrov, K. Sarksian, and N. Skvortsova, Nucl. Fusion 42, 787 (2002).

[6]R. Balescu, Aspects of Anomalous Transport in Plasmas (IoP publishing, Bristol and Philadelphia, 2005).

[7]D. del-Castillo-Negreta, P. Mantica, V. Naulin, and Rasmusen, Nucl. Fusion 48, 75009 (2008).

[8]G. Dif-Pradalier, P. H. Diamond,V. Grandgirard, Y. Sarazin, J. Abiteboul, X. Garbet, Ph. Ghendrih, A. Strugarek, S. Ku, and C. S. Chang, Phys. Rev. E 82, 025401 (2010).

[9]R. Sanchez, D. E. Newman, J.-N. Leboeuf, V. K. Decyk, and B. A. Carreras, Phys. Rev. Lett. 101, 205002 (2008).

[10]D. del-Castillo-Negrete, B. A. Carreras, and V. E. Lynch, Phys. Rev. Lett. 94, 065003 (2005).

[11]M. Schlesinger, G. M. Zaslavsky and J. Klafter Nature, 363, 31 (1993).

[12]R. Metzler and J. Klafter Phys. Reports 339, 1 (2000).

[13]R. Metzler and J. Klafter J. Phys. A: Math. Gen. 37, R161 (2004).

[14]B. B. Mandelbrot The Fractal Geometry of Nature. W. H. Freeman and Company, San Francisco (1982).

[15]A. V. Chechkin, V. Yu. Gonchar and M. Szydlowski Phys. Of Plasmas 9, 78 (2002).

[16]D. del-Castillo-Negrete, B. A. Carreras, and V. E. Lynch, Phys. of Plasmas 11, 3854-3864 (2004).

[17]D. del-Castillo-Negrete, Nonlin. Processes Geophys., 17, 795-807, (2010).

[18]J. Anderson, E. Kim and S. Moradi, Phys. Plasmas 21, 122109 (2014).

[19]B. A. Carreras, C. Hidalgo, E. Sanchez, M. A. Pedrosa, R. Balbin, I. Garcia-Corts, B. van Milligen, D. E. Newman, and V. E. Lynch, Phys. Plasmas 3, 2664 (1996).

[20]B. A. Carreras, B. Ph. van Milligen, C. Hidalgo, R. Balbin, E. Sanchez, I. Garcia-Cortes, M. A. Pedrosa, J. Bleuel, and M. Endler, Phys. Rev. Lett. 83, 3653 (1999). [21]R. Jha, P.K. Kaw, D.R. Kulkarni, J.C. Parikh and ADITYA Team, Phys. of Plasmas 10, 699 (2003).

[22]T. Mizuuchi, V.V. Chechkin, K. Ohashi, E.L. Sorokovoy, A.V. Chechkin, V. Yu. Gonchar, K. Takahashi, S. Kobayashi, K. Na- gasaki, H. Okada, S. Yamamoto, F. Sano, K. Kondo, N. Nishino, H. Kawazome, H. Shidara, S. Kaneko, Y. Fukagawa, Y. Morita, S. Nakazawa, S. Nishio, S. Tsuboi, and M. Yamada, Journal of Nuclear Materials, 337-339 (2005) 332-336.

[23]V. Yu. Gonchar, A. V. Chechkin, E. L. Sorokovoi, V. V. Chechkin, L. I. Grigoreva, and E. D. Volkov, Plasma Physics Reports, Vol. 29, No. 5, pp. 380-390 (2003). Translated from Fizika Plazmy, Vol. 29, No. 5, pp. 413-423 (2003).

[24]K. Burnecki, A. Wylomanàka, A. Beletskii, V. Gonchar and A. Chechkin, Phys. Rev. E. 85 056711 (2012).

[25] K. Burnecki, A. Wylomanàka and A. ONE,10(12): e0145604. (2015).

[26]S. T. Bramwell, Nat. Phys. 5, 443 (2009).

[27]P. Lévy, Theorie de l'Addition des Variables, Gauthier-Villiers, Paris (1937).

[28]Sz. Borak, W. Hädle and R. Weron, Statistical Tools for Finance and Insurance, eds. Springer-Verlag, Berlin, 21-44 (2005), http://ideas.repec.org/p/hum/wpaper/sfb649dp2005-008.html.

[29]J. M. Chambers, C. L. Mallows and B. W. Stuck, A Method for Simulating Stable Random Variables, JASA, 71, 340-344 (1976). [30]R. Weron, J.E. Gentle, W. Härdle and Y. Mori, Computationally intensive Value at Risk calculations in Handbook of Computational Statistics: Concepts and Methods, eds. Springer, Berlin, 911-950 (2004).

[31]I. Pavlyukevich and M. Riedle, Stoch. Anal. Appl. 33, 271 (2015).

[32]Anomalous Transport: Foundations and applications". Edited by R. Klages, G. Radons and I.M. Sokolov (2008 Wiley-VCH, Germany)

[33]D. del-Castillo-Negrete: "Non-diffusive transport in fusion plasmas: fractional diffusion approach." Chapter in proceedings of the "First ITER Summer School: Turbulent Transport in Fusion Plasmas." Edited by S. Benkadda, AIP Conference Proceedings, Vol.1013 (AIP, Melville, New York, 2008).

b) Self-organisation of random oscillators with Levy stable distributions:

A simple mathematical but yet powerful tool to study the dynamics of a many-body interacting systems is the Kuramoto's system of randomly coupled limit cycle oscillators. Over the years, many aspects of the model, including applications cutting across disciplines, from physical and biological to even social modelling, have been considered in the literature [1-11]. A particular application of the model has been in turbulence theory where the model can be employed to examine various aspects of the non-linear dynamics. In a previous work, we developed a predator-prey model of dual populations with stochastic oscillators to examine several important features of the dynamical interplay between the drift wave and zonal flow turbulence in magnetically confined plasmas [12]. The underlying reasoning is that by rewriting the function representing the fluctuation quantities as $f_k = |f_k| \exp(i\theta_k(t) + i \vec{k} \cdot \vec{r})$ and following the typical quadratically nonlinear primitive equations that arise in practice as:

$$\frac{df_k}{dt} + i\omega_k f_k + \sum_{k=k'+k''} M_{k'k''} f_{k'} f_{k''} = 0 \tag{1}$$

we find a phase evolution equation of the Kuramoto form. By including an additive noise term on the RHS of the above equation we can pave the way for use of efficient analytical tools commonly employed in the study of the statistical behaviour of many-body systems. Furthermore, by treating the $M_{k'k''}$ as a random coefficient we can gain insights into properties of multiplicative statistics, albeit with radical simplifications, that are more common in practice e.g. the advective nonlinearities in the Navier-Stokes, MHD, gyrokinetic, and other equations. Thus, statistical theories can be viewed as reduced descriptions of the wealth of information in the true turbulent dynamics, and there has been a wide range of applications based on these statistical treatments e.g Langevin equations for an additive [13] and Stochastic Oscillator model as a multiplicative model [14].

An important shortcoming here is that the main body of work has been with the focus on Gaussian statistical assumptions. Although Gaussian statistics can in certain cases of diffusion in time and space give a good representation of the apparent randomness, in many systems there are processes where the Gaussian approach is inappropriate [15–20]. There is a wealth of experimental and numerical evidence that indicate that turbulent trans- port under some conditions, is non-diffusive [21]. There are several reasons for the possible breakdown of the standard diffusion paradigm which is based on restrictive assumptions including locality, Gaussianity, lack of long range correlations, and linearity. Different physical mechanisms can generate situations where e.g., locality and Gaussianity may be incorrect assumptions for under- standing transport. For example, interactions with external fluctuations may introduce long-range correlations and/or anomalously large particle displacements [22, 23]. The source of the external fluctuations could be that not all relevant physics is taken into account such as coherent modes or other non-linear mechanisms. In addition, turbulence intermittency is characterized by patchy spatial structures that are bursty in time and coupling to these modes introduces long range correlations and/or Lévy distributed noise characteristics [24]. The probability density functions (PDF) of intermittent events often show unimodal structure with "elevated" tails that deviate from Gaussian predictions. The experimental evidence of the wave-number spectrum characterised by power laws over a wide range of wave-numbers can be directly linked to the values of Lévy index α of the PDFs of the underlying turbulent processes. The Lévy-type turbulent random processes and related anomalous diffusion phenomena have been observed in a wide variety of complex systems such as semiconductors, glassy materials, nano-pores, biological cells, and epidemic spreading. The problem of finding a proper kinetic description for such complex systems is a challenge. The pedagogical applications of simplified models such as Kuramoto model of random oscillators are particularly beneficiary in understanding these non-local and non-Gaussian aspects of dynamics in many-body interacting systems.

In this work, we applied the model to study the impact of both additive and multiplicative non-Gaussian forcing which has not been considered up to now. In particular, we are interested to show the dominant impact of singular events with high amplitude on the long term collective behaviours, and to illustrate the limitations of the Gaussian assumptions in these non-linearly coupled systems. We hope to start a wider discussion on the features that can be expected in the case of strange kinetics capturing new ways of synchronization with anomalous processes opening for new fields of application such as that of transport and turbulent dynamics.

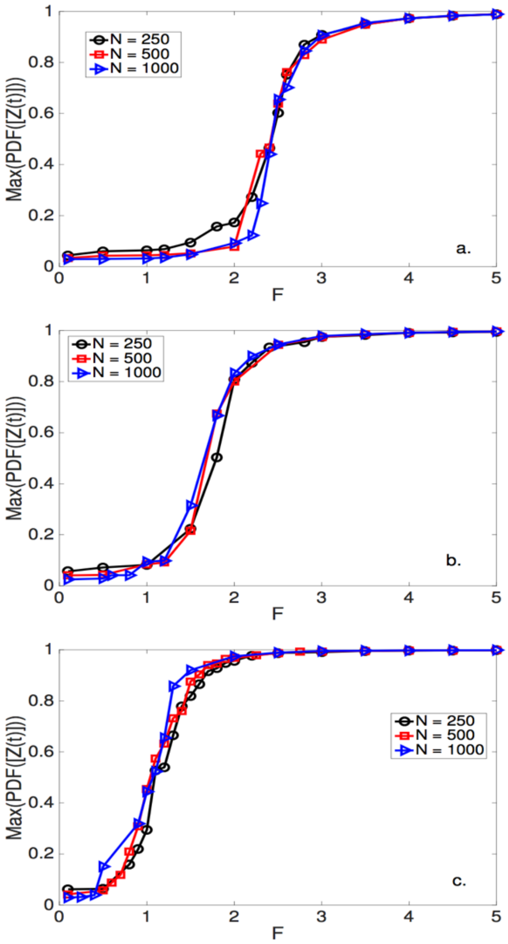

FIG. 2. The maximum of the $PDF([Z(t)])$ as functions of the control parameters F for $\alpha = 2$ (a), $\alpha = 1.5$ (b) and $\alpha = 1.2$ (c). In each figure a scan over different populations are performed with $N = 250$ (black line with circles), $N = 500$ (red line with squares) and $N = 1000$ (blue line with triangles).

THE NUMERICAL SET UP

The dynamics of the phases of the oscillators are described by coupled first order differential equations:

$$\dot{\theta}_j(t) = \omega_j + (2\pi)^{-1} \sum_{i=1}^{N} J_{ij} sin(\theta_i - \theta_j), \quad (j = 1, ..., N), \qquad (2)$$

the oscillator which is assumed to be distributed according to a Gaussian distribution J_{ij} is the strength of the interactions between oscillators ith and jth and are assumed to be random constants distributed according to an α-stable distribution $S(\alpha,\beta,\sigma,\mu)$ with characteristic exponent $0 < \alpha \leq 2$, skewness β, scale σ and location μ [12, 25–31]. Here we chose β=0, μ=0, and $\sigma=F/(N\sqrt{2})$ where F is the control parameter as in Ref. [6]. Moreover, we assume positive and symmetric coupling, i.e. $J_{ij} > 0$ and $J_{ij} = J_{ji}$ respectively. The α-stable distributions are a general class of distributions which also include Gaussian (α = 2) and Lorentzian (α = 1) distributions.

In this work, the numerical integration is performed using the Runge-Kutta 4th order scheme with time step- ping length $\delta t = 2\pi \times dt$ where dt is the optimum time interval varying for each integration while the sampling time step is $\Delta t = 0.01$. The numerical integration is per- formed for the incoherent initial set with θ_j (t = 0) taken to be positive Gaussian distributed random values for an ensemble of N oscillators. Here we employ an average of J_{ij} over a number of different realisations denoted by $N_S = 10$, hereafter referred to as "samples". In the present study, the time span considered is of the order of $2\pi \times 10$. This time span is found to be long enough for the system to reach a steady-state and the numerical noise due to the finite size effects are absent.

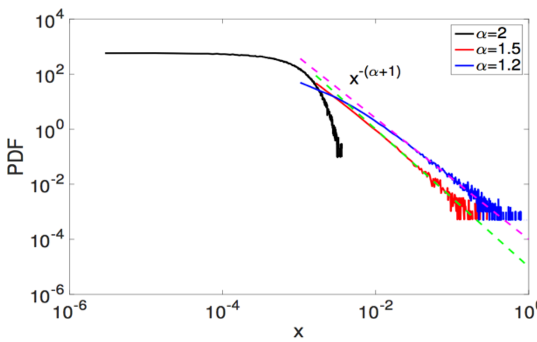

FIG. 1. PDFs of the coupling strength parameter J_{ij} according to α-stable distribution with $\alpha = 2$ (black), 1.5 (red), 1.2 (blue). Dotted lines represent the corresponding $x^{-(1+\alpha)}$ fits of the PDFs.

LÉVY COUPLED OSCILLATORS

We have performed numerical integrations for different values of the fractality index α, e.g. α = 2, 1.5, 1.2. Figure 1 illustrates the normalised PDFs of the coupling strengths J_{ij} for the selected αs. Here as we change the index α from 2 to 1.2 the tails of the distribution become heavier, indicating the further increase in probability of couplings between oscillations with higher strength even at low F values. A $x^{-(1+\alpha)}$ power decay fit which is typical of α-stable distributions confirms the proper scaling of the Lévy stable process generated by the random generator used here, (see Fig. 1). An analytic expression for the order parameter $Z(t) = \Sigma^{N}_{j=1} \exp(i\theta_j)/N$ was derived by Kuramoto that de- scribes the quality of the synchronisation of the ensemble of oscillators with $0 \leq Z \leq 1$. Where Z = 0 corresponds to a complete a-synchronised state while Z = 1 corresponds to a total synchronised state. We have calculated the values of the order parameter averaged over N_s samples as well as for the various cases considered. Figures 2(a-c) show the order parameter Z(t) as a function of F for different number of oscillator populations N = 250, 500, 1000, with different values of α-stable distribution index α = 2, 1.5, 1.2. As can be seen in Fig. 2 (a-c) for low values of control parameters i.e. $F \leq 2$ the phases are a-synchronised with $[Z(t)] \approx 0$. As the control parameters increase beyond this threshold the phases bifurcate from an a-synchronised to a synchronise state. The threshold where the populations change from a synchronised to a-synchronised state, in agreement with the previous reports (see Refs. [5, 6]), is found to be independent of the number of considered oscillators. In the following thus, we fixed the number of populations to N = 250.

Figure 3 compares the computed values for N = 250 between the different αs. As can be seen in these figures, a bifurcation to a synchronised state occurs as F is increased beyond a critical value F_c which holds for all fractality index values considered in the α-stable distribution. However, there is a significant shift to lower values of the criticality parameter F_c as α is decreased from 2, where 2 corresponds to the Gaussian distribution. This indicates that the extreme events from the tails of the distribution can provide a faster synchronisation of the coupled oscillators, and as the tail gets heavier by moving from α = 1.5 to 1.2, F_{dc} is shifted to even lower values.

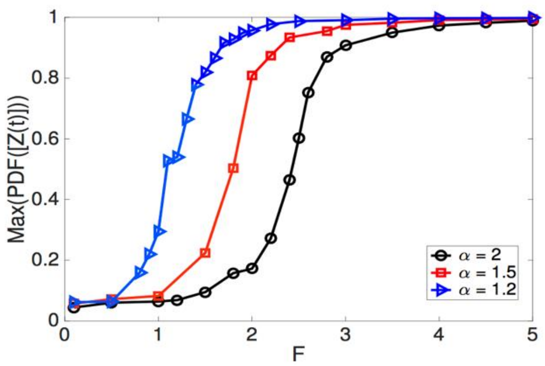

FIG. 3. The comparison of the maximum of the $PDF([Z(t)])$ as functions of the control parameters F for $\alpha = 2$ (black line with circles), $\alpha = 1.5$ (red line with squares) and $\alpha = 1.2$ (blue line with triangles). Here, N=250.

In conclusions, we find that extreme events in a non-linear coupled system such as Kuramoto model, govern the long-term behaviour of such systems. In a complex system of a realistic turbulence state therefore, the impact of such events has to be analysed and simplistic assumptions on the statistics of underlying fluctuations cannot represent their long-term behaviour.

[1] Y. Kuramoto, Chemical oscillations, Waves and Turbulence (Springer, Berlin) (1984).
[2] S. H. Strogatz, Physica D 143 1 (2000).
[3] A. J. Acebrón, L. L. Bonilla, C. J. P. Vicente, F. Ritort and R. Spigler, Rev Mod Phys 77 137 (2005).
[4] A. T. Winfree, The Geometry of Biological Time, Springer, New York (1980).
[5] H. Daido, Prog. Theor. Phys. 77, 622 (1987).
[6] H. Daido, Phys. Rev. Lett. 68, 1073 (1992).
[7] J. D. Crawford, J. Stat. Phys. 74, 1047 (1994).
[8] H. Daido, Phys. Rev. E 61, 2145 (2000).
[9] S. H. Strogatz, Sync: The Emerging Science of Spontaneous Order, Hyperion, New York (2003).
[10] H. Hong and S. H. Strogatz, Phys. Rev. Lett. 106, 054102 (2011).
[11] B. Sonnenschein and L. Schimansky-Geier, Phys. Rev. E 88, 052111 (2013).

[12] S. Moradi, J. Anderson and O. Gurcan, Phys. Rev. E 92 062930 (2015).
[13] P. Langevin, Sur la théorie du mouvement brownien. C. R. Acad. Sci. 146 530 (1908).
[14] R. H. Kraichnan, Variational method in turbulence theory. Phys. Rev. Lett. 42 1263 (1979).
[15] J. Klafter and I. M. Sokolov Phys. World bf 8, 29 (2005).
[16] R. Metzler and J. Klafter Phys. Reports 339, 1 (2000).
[17] E. W. Montroll and H. Scher J. Stat. Phys. 9, 101 (1973).
[18] G. M. Zaslavsky Physica D 76 110 (1994).
[19] S. C. Kou and X. Sunney Xie Phys. Rev. Lett. 93, 180603 (2004).
[20] E. W. Montroll and J. T. Bendler J. Stat. Phys. 34, 129 (1984).
[21] K. Ida, Z. Shi, H.J. Sun, S. Inagaki, K. Kamiya, J.E. Rice, N. Tamura, P.H. Diamond, G. Dif-Pradalier, X.L. Zou, K. Itoh, S. Sugita, O.D. Gurcan, T. Estrada, C. Hidalgo, T.S. Hahm, A. Field, X.T. Ding, Y. Sakamoto, S. Oldenburger, M. Yoshinuma, T. Kobayashi, M. Jiang, S.H. Hahn, Y.M. Jeon, S.H. Hong, Y. Kosuga, J. Dong and S.-I. Itoh, Nucl. Fusion 55, 013022 (2015).
[22] S. Moradi, D. del-Castillo-Negrete and J. Anderson, Phys. Plasmas 23 090704 (2016).
[23] J. Anderson, E. Kim and S. Moradi, Phys. Plasmas 21, 122109 (2014).
[24] P. Lévy, Theoriedel' Addition des Variables, Gauthier-Villiers, Paris (1937).
[25] Sz. Borak, W. Härdle and R.Weron, Sta- tistical Tools for Finance and Insurance, eds. Springer-Verlag, Berlin, 21-44 (2005), http://ideas.repec.org/p/hum/wpaper/sfb649dp2005- 008.html.
[26] J. M. Chambers, C. L. Mallows and B. W. Stuck, A Method for Simulating Stable Random Variables, JASA, 71, 340-344 (1976).
[27] R.Weron, J.E.Gentle, W. Härdle and Y.Mori, Computationally intensive Value at Risk calculations in Handbook of Computational Statistics: Concepts and Methods, eds. Springer, Berlin, 911-950 (2004).
[28] S. Kirkpatrick and D. Sherrington, Phys. Rev. B 17 4384 (1978).
[29] H. Sakaguchi, Prog. Theor. Phys. 79, 39 (1988).
[30] J. A. Acebrón, A. Perales and R. Spigler, Phys. Rev. E 64, 016218 (2001a).
[31] S. Moradi and J. Anderson, Phys. Plasmas 23, 052310 (2016).
[32] D. J. Higham, Society for Industrial and Applied Mathematics (SIAM) Review, Vol. 43, No. 3, 525-546 (2001).

4. Valorisation/Diffusion (including Publications, Conferences, Seminars, Missions abroad...)

- Peer-reviewed Journals:

1) S. Moradi, B. Teaca, The role of phase dynamic in Burger's Turbulence, in preparation.

2) S. Moradi, A random walk model for the motion of particles in multi-scaled systems, submitted.

3) S. Moradi, and J. Andersson: Self-organisation of random oscillators with Levy stable distributions, under review in PRE.

4) S. Moradi, D. del-Castillo-Negreta and J. Andersson: Charge particle motion in the presence of non-Gaussian, Levy, electrostatic fluctuations, Phys. Plasmas, 23 090704 (2016).

- Conference proceedings:

1) G. Flachetto et al., EuroFusion Integrated Modelling (EU-IM) capabilities and selected physics applications. Proceedings 26th IAEA Fusion Energy Conference, Kyoto (Japan), 17-22 October 2016.

2) T. Tala, A. Salmi, M. Maslov, L. Meneses, S. Menmuir, S. Moradi, S. Mordijk, V. Naulin, H. Nordman, J.J. Rasmussen, A. Sips, J. Svenssen, C. Bourdelle, M. Tsalas, H. Weisen, L. Giacomelli, C. Giroud, R. Gomes, J. Hilleshiem, A. E. Jarvinen, C. Maggi, and P. Mantica, Density Peaking in JET - Driven by Fuelling or Transport?. Proceedings 26th IAEA Fusion Energy Conference, Kyoto (Japan), 17-22 October 2016.

- Poster presentations:

S. Moradi and J. Anderson: Phase dependent advection-diffusion in drift wave - zonal flow turbulence. 43th European Physical Society meeting on Plasma Physics, Leuven, Belgium, 4-8 July 2016.

S. Moradi and J. Anderson: The role of phase dynamics in a stochastic model of a passively advected scalar. 21st Joint EU-US Transport Task Force Meeting, Leysin, Switzerland, from 4-8 September 2016.

- Conferences Talks:

Self-organisation of stochastic oscillators in a predator-prey model, Belgian Physical Society Meeting, Gent, Belgium (18th May 2016).

Self-organisation of stochastic oscillators in a predator-prey model, 9th CHAOS conference, London, UK (23-26th May 2016).

-Missions abroad:

Participation in EuroFusion-JET 2016, C36 experimental campaign as Session Leader Trainee and transport modeling expert, Culham Center for Fusion Energy (CCFE), Culham, UK, 13^{th} – 24^{th} March and 28^{th} March to 9^{th} April 2016.

5. Future prospects for a permanent position in Belgium

I have obtained a position as a Fusion researcher at the Laboratory for Plasma Physics - LPP-ERM/KMS Royal Military Academy - Department of Physics - EUROfusion Consortium Member.

6. Miscellaneous

Supervision:

1. Master's thesis entitled "Global vs local gyro-kinetic studies of core micro-instabilities in JET hybrid discharges with ITER like wall".